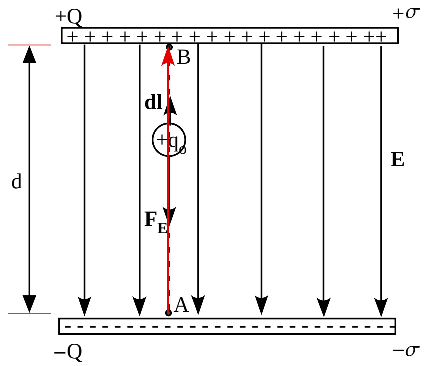
# Electric Potential

The quantity of electric potential is a very useful quantity in analyzing a wide variety of problems in electricity. As we will see it will provides us with a simpler method of calculating electric potential energy and electric fields.

To obtain an equation for the electric potential let's begin by finding the work done by the electric force on a positive charge as it moves in a uniform E-field.




$$w_{AB} = \int_A^B \vec{F}_E \bullet d\vec{\ell} = \vec{F}_E \bullet \int_A^B d\vec{\ell} = \vec{F}_E \bullet \vec{\ell} = -F_E q_o d$$

$$\boxed{w_{AB} = -q_o E d}$$

Since the electric force is conservative, then

$$w_{AB} = -\Delta U_{AB}$$

$$\boxed{\Delta U_{AB} = q_o E d}$$

a) If $q_o > 0$, then $\Delta U_{AB} > 0$ and $W_{AB} < 0$.
b) If $q_o < 0$, then $\Delta U_{AB} < 0$ and $W_{AB} > 0$.

In general, U increases if a charge moves in opposite direction of electric force and decreases if it moves in same direction.

Note that $\Delta U_{AB}$ is the same value regardless of the path taken between A and B. However, $\Delta U_{AB}$ is proportional to the charge $q_o$. If you double $q_o$, then you also double $\Delta U_{AB}$.

| $q_o$ | $\Delta U_{AB}$ | $\Delta U_{AB}/ q_o$ |
|---|---|---|
| q | qEd | Ed |
| 2q | 2qEd | Ed |
| 10q | 10qEd | Ed |

Thus, the quantity $\Delta U_{AB}/ q_o$ is a constant value that is independent of the charge $q_o$. We define the quantity $\Delta U_{AB}/ q_o$ as the Electric Potential Difference.

$$\boxed{\Delta V_{AB} = \frac{\Delta U_{AB}}{q_o}}$$ Electric Potential Difference

$$V_B - V_A = \frac{U_B}{q_o} - \frac{U_A}{q_o}$$

$$V_A = \frac{U_A}{q_o} \quad \text{(Electric Potential at point A)}$$

$$V_B = \frac{U_B}{q_o} \quad \text{(Electric Potential at point B)}$$

$$\boxed{V = \frac{U}{q}}$$ Electric Potential

$$\boxed{U = qV}$$ Electric Potential Energy

Therefore, if you know the electric potential at some point in space, then you can always compute the electric potential energy of a charge *q* placed at that point.

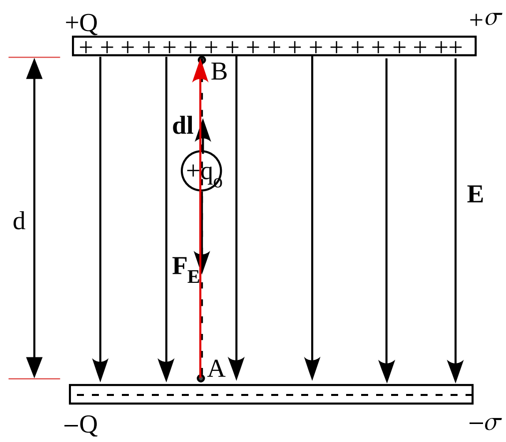
Going back to our definition of Electric Potential Difference,

$$\Delta V_{AB} = \frac{\Delta U_{AB}}{q_o} = \frac{-W_{AB}}{q_o} = \frac{-\int_A^B \vec{F}_E \bullet d\vec{\ell}}{q_o} = \frac{-\int_A^B q_o \vec{E} \bullet d\vec{\ell}}{q_o}$$

$$\boxed{\Delta V_{AB} = V_B - V_A = -\int_A^B \vec{E} \bullet d\vec{\ell}}$$ Definition of Potential Difference

1. The SI unit of potential is the volt (V): 1 V = 1 J/C
2. Potential is a scalar quantity.
3. Only changes in potential have a physical meaning.
4. ΔV is a property of the **E**-field and independent of the charges placed in the **E**-field.
5. $\Delta V_{AB} = \frac{W_{ext\ AB}}{q_o}$ = work done per unit charge by an external agent in moving a charge $q_o$ from A to B against the electric force w/o changing its kinetic energy.

## Potential Difference in a Uniform E-field




$$\Delta V_{AB} = V_B - V_A = -\int_A^B \vec{E} \bullet d\vec{\ell} = -\vec{E} \bullet \int_A^B d\vec{\ell} = -\vec{E} \bullet \vec{\ell} = Ed$$

Or also,

$$\Delta V_{AB} = V_B - V_A = \frac{\Delta U_{AB}}{q_o} = \frac{q_o E d}{q_o} = Ed$$

1. Point B is at a higher potential
2. **E**-field always points in the direction in which V decreases.
3. *Positive charges* move from a ***high potential*** to a ***low potential***.
4. *Negative charges* move from a ***low potential*** to a ***high potential***.
5. Since $\Delta V_{AB} = \frac{\Delta U_{AB}}{q_o}$, potential difference is a measure of how much PE a charge can gain/lose.