

DIRECTIONS To receive full credit, you must provide complete legible solutions to the following problems in the space provided. Be sure to supply all the necessary steps that lead to your answers.

1. Determine the singular points of the given differential equation, then classify each singular point, as regular or irregular, then for each regular singular point determine the form of the solution of the DE. **Be sure to justify your answer**

a) $(x^3 + 4x)y'' - 2xy' + 6y = 0$

2. $x = 0$ is a regular singular point of the given differential equation. Determine the form of

$$x^2 y'' + (3x + x^2)y' + y = 0$$

the solution of the DE. **Be sure to justify your answer.** Do not solve the equation.

3. The method of undetermined coefficient is used to solve the equation

$$xy'' + 4y' - xy = 0$$

about the regular singular point $x = 0$, and the result of the substitution of

$$y = \sum_{n=0}^{\infty} c_n x^{n+r} \text{ is given below.}$$

Find a fundamental set of solutions to the DE.

$$\begin{aligned} & [r(r-1) + 4r]c_0 x^{r-1} + [r(r+1) + 4(r+1)]c_1 x^r \\ & + \sum_{k=0}^{\infty} \{(k+r+1)(k+r+4)c_{k+1} - c_{k-1}\} x^{k+r} = 0 \end{aligned}$$