Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1. Determine if the given set forms a Basis for $R^3$.

a. $\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix},$

b. $\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -4 \\ -2 \end{bmatrix}$

2. Find a basis for the set of vectors in $R^3$ in the plane $x + 2y + z = 0$.

3. Find a Basis for the Nul(A) and Col(A) of the given matrix.

$$A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 0 & -5 \\ 0 & -5 & 7 \\ -5 & 7 & -2 \end{bmatrix}$$

4. Let H = Span{$v_1$, $v_2$, $v_3$} and K = Span{$u_1$, $u_2$, $u_3$} where

$v_1 = (1,1,2),\ v_2 = (-1,2,1),\ v_3 = (0,1,1)$

$u_1 = (1,-1,1),\ u_2 = (-1,1,1),\ u_3 = (-3,3,-1)$

Find Basis for H, K and H+K.

5. Show that $\{t, \sin t, \cos 2t, \sin t \cos t\}$ s a linearly independent set of functions defined on R. Start by assuming that $c_1 t + c_2 \sin t + c_3 \cos 2t + c_4 \sin t \cos t$ Equation (5) must hold for all real t, so choose several specific values of t (say, t=0, 0.1, 0.2) until you get a system of enough equations to determine that all the $c_j$ must be zero.

5. Find an explicit description of Nul(A) by listing vectors that span the null space.

$$A = \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 1 & 4 & -2 \end{bmatrix}$$