

Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1. Let $D = \{d_1, d_2\}$ and $B = \{b_1, b_2\}$ be bases for vector spaces V and W , respectively. Let $T : V \rightarrow W$ be a linear transformation with the property that $T(d_1) = 2b_1 - 3b_2$, $T(d_2) = -4b_1 + 5b_2$. Find the matrix for T relative to D and B .

2. Let $T : \mathbb{P}_2 \rightarrow \mathbb{P}_4$ be the transformation that maps a polynomial $\mathbf{p}(t)$ into the polynomial $p(t) + t^2 p(t)$
 - a. Find the image of $p(t) = 2 - t + t^2$
 - b. Show that T is a linear transformation.
 - c. Find the matrix for T relative to the bases $\{1, t, t^2\}$ and $\{1, t, t^2, t^3, t^4\}$

3. Let $B = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ be a basis for a vector space V . Find $T(3\mathbf{b}_1 - 4\mathbf{b}_2)$ when T is a linear transformation from V to V whose matrix relative to B is

$$[T]_B = \begin{bmatrix} 0 & -6 & 1 \\ 0 & 5 & -1 \\ 1 & -2 & 7 \end{bmatrix}$$

4. Find the B -matrix for the transformation $\mathbf{x} \mapsto A\mathbf{x}$ when $B = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$.

$$A = \begin{bmatrix} -14 & 4 & -14 \\ -33 & 9 & -31 \\ 11 & -4 & 11 \end{bmatrix}, \mathbf{b}_1 = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \mathbf{b}_3 = \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix}$$

5. Let T be a transformation whose standard matrix is given below. Find a Basis for \mathcal{R}^3

with the property that $[T]_B$ is diagonal.

$$A = \begin{bmatrix} 2 & 0 & -2 \\ 1 & 3 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$